BREAKTHROUGHS ON THE DARK MATTER ISSUE

ANTONIO MASIERO and FRANCESCA ROSATI
SISSA, Via Beirut 2-4, I-34013 Trieste, ITALY
INFN, sezione di Trieste, Padriciano 99, I-34012 Trieste, ITALY
E-mail: masiero@sissa.it, rosati@sissa.it

ABSTRACT

Last year observations had a profound impact on our views on the amount and nature of dark matter in the universe. We give a brief review of the recent history of dark matter models beyond the pure cold dark matter universe. In view of the most recent cosmological data, we then go on to discuss models with a positive cosmological constant. Finally we explicitly analyse a class of particle physics models for a dynamical cosmological component with negative pressure ("quintessence"), in the context of supersymmetric theories.

1. Introduction

1.1. The Universe with Λ

The year 1998 has witnessed major changes in our perspectives on the amount and nature of the different components of dark matter (DM) in the universe.

The indications for a presently accelerating universe coming from redshift-distance measurements of High-Z Supernovae Ia (SNe Ia)[1,2], combined with cosmic microwave background (CMB) data[3] and cluster mass distribution[4] seem to favour models with a non-vanishing cosmological constant Λ. Indeed the energy density contributed by Λ should roughly be twice as much as the enegy density of the matter of the whole universe, thus leading to a flat universe (with $\Omega_{total} = 1$) composed by 1/3 of matter and the remaining 2/3 of cosmological constant or vacuum energy.

If these results have greatly excited the community of astrophysicists and cosmologists, they have not left indifferent the particle physicists. Indeed, most of the people belonging to latter group used to think that the famous problem of the smallness of the cosmological constant would have eventually found a solution with the discovery of some symmetry that could make it vanishing. On the contrary, the 1998 observational evidence seems to indicate that Λ, far from being zero, could instead constitute the major source of energy of a flat (critical) universe.

But what is actually Λ? Last year data support the view that the universe is presently dominated by a smooth component with effective negative pressure; this is in fact the most general requirement in order to explain the observed accelerated expansion. The most straightforward candidate for that is, of course, a "true" cosmological constant[5], but a plausible alternative that has recently received a great deal

of attention is a dynamical vacuum energy given by a scalar field rolling down its potential. A cosmological scalar field, depending on its dynamics, can easily fulfill the condition of an equation of state $w = p/\rho$ between -1 (which corresponds to the cosmological constant case) and 0 (that is the equation of state of matter). Since it is useful to have a short name for the rather long definition of this dynamical vacuum energy, we follow the literature in calling it briefly "quintessence"[6].

The major bulk of this talk will be devoted to the discussion of the "quintessence" possibility in the context of supersymmetric (SUSY) theories. But before plunging into this, let us remind the readers that the abovementioned data should be taken with some caution.

1.2. The alternatives

It is indeniably impressive that having three sets of independent data depending on two parameters – the fraction densities in matter and cosmological constant, Ω_m and Ω_Λ – we get an overconstrained system with a unique solution (what has been called with ironical emphasis "cosmic concordance"[7]). However, one should not underestimate the fact that for some of the measurements – for instance the position of the first acoustic peak in the CMB and the study of the systematics in the SNe Ia – we are still at a preliminary level. While waiting for further observational evidence in favour of cosmologies based on cold DM (CDM) and quintessence (also called QCDM models), one should then anyway keep an open eye on the alternative solutions to the dark matter problem which have emerged after the crisis of the pure CDM standard model.

In the pure CDM model, almost all of the energy density needed to reach the critical one (the remaining few percent being given by the baryons) was provided by cold dark matter alone. However, some observational facts (in particular the results of COBE) put this model into trouble, showing that it cannot correctly reproduce the power spectrum of density perturbations at all scales. At the same time it became clear that some amount of CDM was needed anyway in order to obtain a successful scheme for large scale structure formation.

A popular option is that of a flat universe realized with the total energy density mostly provided by two different matter components, CDM and hot DM (HDM) in a convenient fraction. These models, which have been called mixed DM (MDM)[8], succeeded to fit the entire power spectrum quite well, although – having $\Omega_m = \Omega_{tot} = 1$ – they obviously can't account for the most recent data.

Another interesting possibility for improving CDM models consists in the introduction of some late time decaying particle[9]. The injection of non-thermal radiation due to such decays and the consequent increase of the horizon length at the equivalence time could lead to a convenient suppression of the excessive power at small scales (hence curing the major desease of the pure CDM standard model). As ap-

pealing as this proposal may be from the cosmological point of view, its concrete realization in particle physics models meets several difficulties. Indeed, after considering cosmological and astrophysical bounds on such late decays, it turns out that only few candidates survive as viable solutions (for a recent analysis in the context of SUSY extensions of the SM with or without R parity see Ref.[10]).

Another route which has been followed in the attempt to go beyond the pure CDM proposal is the possibility of having some form of warm DM (WDM). The implementation of this idea is quite attractive in SUSY models where the breaking of SUSY is conveyed by gauge interactions instead of gravity (these are the so-called gauge mediated SUSY breaking (GMSB) models). In these schemes the gravitino mass loses its role of fixing the typical size of soft breaking terms and we expect it to be much smaller than in the more traditional supergravity models. The gravitino in GMSB theories can behave as a WDM candidate. Unfortunately, critical universes with pure WDM are known to suffer from serious troubles[11]. It has been shown that even variants of the pure WDM models (with an additional HDM component or with a non-vanishing cosmological constant) still exhibit various problems in correctly reproducing the large scale structure data[12].

Finally, the alternative to pure CDM which succeeded to fit the whole power spectrum as well as the MDM models was provided by the so-called xCDM models[13]. In those schemes, CDM is accompanied with a cosmological constant-like contribution (named x) adding up to reach the critical energy density. In view of the recent observations that we presented above, it is clear that xCDM models today enjoy the major success. We now turn to discuss these schemes in the context of SUSY theories[14].

2. Supersymmetry and Quintessence

As already said in the introduction, the 1998 indications for an accelerating universe have recently drawn a great deal of attention on cosmological models with $\Omega_m \sim 1/3$ and $\Omega_\Lambda \sim 2/3$. More generally, the rôle of the cosmological constant in accelerating the universe expansion could be played by any smooth component with negative equation of state $p_Q/\rho_Q = w_Q \lesssim -0.6$[6,15], as in the so-called "quintessence" models (QCDM)[6], otherwise known as xCDM models[13].

2.1. Scalar field cosmology

A natural candidate for quintessence is given by a rolling scalar field Q with potential $V(Q)$ and equation of state

$$w_Q = \frac{\dot{Q}^2/2 - V(Q)}{\dot{Q}^2/2 + V(Q)} \,,$$

which – depending on the amount of kinetic energy – could in principle take any

value from -1 to $+1$. The study of scalar field cosmologies has shown[16,17] that for certain potentials there exist attractor solutions that can be of the "scaling" [18,19,20] or "tracker"[21,22] type; that means that for a wide range of initial conditions the scalar field will rapidly join a well defined late time behavior.

If $\rho_Q \ll \rho_B$, where ρ_B is the energy density of the dominant background (radiation or matter), the attractor can be studied analytically.

In the case of an exponential potential, $V(Q) \sim \exp(-Q)$ the solution $Q \sim \ln t$ is, under very general conditions, a "scaling" attractor in phase space characterized by $\rho_Q/\rho_B \sim \text{const}$[18,19,20]. This could potentially solve the so called "cosmic coincidence" problem, providing a dynamical explanation for the order of magnitude equality between matter and scalar field energy today. Unfortunately, the equation of state for this attractor is $w_Q = w_B$, which cannot explain the acceleration of the universe neither during RD ($w_{rad} = 1/3$) nor during MD ($w_m = 0$). Moreover, Big Bang nucleosynthesis constrain the field energy density to values much smaller than the required $\sim 2/3$ [17,19,20].

If instead an inverse power-law potential is considered, $V(Q) = M^{4+\alpha}Q^{-\alpha}$, with $\alpha > 0$, the attractor solution is $Q \sim t^{1-n/m}$, where $n = 3(w_Q + 1)$, $m = 3(w_B + 1)$; and the equation of state turns out to be $w_Q = (w_B\,\alpha - 2)/(\alpha + 2)$, which is always negative during MD. The ratio of the energies is no longer constant but scales as $\rho_Q/\rho_B \sim a^{m-n}$ thus growing during the cosmological evolution, since $n < m$. ρ_Q could then have been safely small during nucleosynthesis and have grown lately up to the phenomenologically interesting values.These solutions are then good candidates for quintessence and have been denominated "tracker" in the literature [17,21,22].

The inverse power-law potential does not improve the cosmic coincidence problem with respect to the cosmological constant case. Indeed, the scale M has to be fixed from the requirement that the scalar energy density today is exactly what is needed. This corresponds to choosing the desired tracker path. An important difference exists in this case though. The initial conditions for the physical variable ρ_Q can vary between the present critical energy density ρ^0_{cr} and the background energy density ρ_B at the time of beginning[22] (this range can span many tens of orders of magnitude, depending on the initial time), and will anyway end on the tracker path before the present epoch, due to the presence of an attractor in phase space[21,22]. On the contrary, in the cosmological constant case, the physical variable ρ_Λ is fixed once for all at the beginning. This allows us to say that in the quintessence case the fine-tuning issue, even if still far from solved, is at least weakened.

A great effort has recently been devoted to find ways to constrain such models with present and future cosmological data in order to distinguish quintessence from Λ models[23,24]. An even more ambitious goal is the partial reconstruction of the scalar field potential from measuring the variation of the equation of state with increasing redshift[25].

On the other hand, the investigation of quintessence models from the particle

physics point of view is just in a preliminary stage and a realistic model is still missing (see for example Refs. [26,27,28,29]). There are two classes of problems; the construction of a field theory model with the required scalar potential and the interaction of the quintessence field with the standard model (SM) fields[30]. The former problem was already considered by Binétruy[26], who pointed out that scalar inverse power law potentials appear in supersymmetric QCD theories (SQCD)[31] with N_c colors and $N_f < N_c$ flavors. The latter seems the toughest. Indeed the quintessence field today has typically a mass of order $H_0 \sim 10^{-33}$eV. Then, in general, it would mediate long range interactions of gravitational strength, which are phenomenologically unacceptable.

In the remaining part of the talk, we will address in more details these problems in the framework of SQCD, following the work done in Ref.[14].

2.2. *Susy QCD*

As already noted by Binètruy[26], supersymmetric QCD theories with N_c colors and $N_f < N_c$ flavors[31] may give an explicit realization of a model for quintessence with an inverse power law scalar potential. The remarkable feature of these theories is that the superpotential is exactly known non-perturbatively. Moreover, in the range of field values that will be relevant for our purposes (see below) quantum corrections to the Kähler potential are under control. As a consequence, we can study the scalar potential and the field equations of motion of the full quantum theory, without limiting ourselves to the classical approximation.

The matter content of the theory is given by the chiral superfields Q_i and $\overline{Q}_i$ ($i = 1 \ldots N_f$) transforming according to the N_c and $\overline{N}_c$ representations of $SU(N_c)$, respectively. In the following, the same symbols will be used for the superfields Q_i, $\overline{Q}_i$, and their scalar components.

Supersymmetry and anomaly-free global symmetries constrain the superpotential to the unique *exact* form

$$W = (N_c - N_f) \left(\frac{\Lambda^{(3N_c - N_f)}}{\det T} \right)^{\frac{1}{N_c - N_f}} \tag{1}$$

where the gauge-invariant matrix superfield $T_{ij} = Q_i \cdot \overline{Q}_j$ appears. Λ is the only mass scale of the theory. It is the supersymmetric analogue of Λ_{QCD}, the renormalization group invariant scale at which the gauge coupling of $SU(N_c)$ becomes non-perturbative. As long as scalar field values $Q_i, \overline{Q}_i \gg \Lambda$ are considered, the theory is in the weak coupling regime and the canonical form for the Kähler potential may be assumed. The scalar and fermion matter fields have then canonical kinetic terms, and the scalar potential is given by

$$V(Q_i, \overline{Q}_i) = \sum_{i=1}^{N_f} \left(|F_{Q_i}|^2 + |F_{\overline{Q}_i}|^2 \right) + \frac{1}{2} D^a D^a \tag{2}$$

where $F_{Q_i} = \partial W/\partial Q_i$, $F_{\overline{Q}_i} = \partial W/\partial \overline{Q}_i$, and

$$D^a = Q_i^\dagger t^a Q_i - \overline{Q}_i t^a \overline{Q}_i^\dagger \ . \tag{3}$$

The relevant dynamics of the field expectation values takes place along directions in field space in which the above D-term vanish, *i.e.* the perturbatively flat directions $\langle Q_{i\alpha}\rangle = \langle \overline{Q}^\dagger_{i\alpha}\rangle$, where $\alpha = 1 \cdots N_c$ is the gauge index. At the non-perturbative level these directions get a non vanishing potential from the F-terms in (2), which are zero at any order in perturbation theory. Gauge and flavor rotations can be used to diagonalize the $\langle Q_{i\alpha}\rangle$ and put them in the form

$$\langle Q_{i\alpha}\rangle = \langle \overline{Q}^\dagger_{i\alpha}\rangle = \begin{array}{ll} q_i \delta_{i\alpha} & 1 \leq \alpha \leq N_f \\ 0 & N_f \leq \alpha \leq N_c \end{array} .$$

Along these directions, the scalar potential is given by

$$\begin{aligned} v(q_i) \;\; &\equiv \;\; \langle V(Q_i, \overline{Q}_i)\rangle = 2\, \frac{\Lambda^{2a}}{\prod_{i=1}^{N_f} |q_i|^{4d}} \left(\sum_{j=1}^{N_f} \frac{1}{|q_j|^2} \right) , \\ a \;\; &= \;\; \frac{3N_c - N_f}{N_c - N_f}, \qquad d = \frac{1}{N_c - N_f}. \end{aligned}$$

In the following, we will be interested in the cosmological evolution of the N_f expectation values q_i, given by

$$\langle \ddot{Q}_i + 3H\dot{Q}_i + \frac{\partial V}{\partial Q_i^\dagger}\rangle = 0 \ \ , \ i = 1, ..., N_f \ .$$

In Ref.[26] the same initial conditions for all the N_f VEV's and their time derivatives were chosen. With this very peculiar choice the evolution of the system may be described by a single VEV q (which we take real) with equation of motion

$$\ddot{q} + 3H\dot{q} - g\frac{\Lambda^{2a}}{q^{2g+1}} = 0 \ , \qquad\qquad g = \frac{N_c + N_f}{N_c - N_f} \ , \tag{4}$$

thus reproducing exactly the case of a single scalar field Φ in the potential $V = \Lambda^{4+2g}\Phi^{-2g}/2$ considered in Refs.[16,17,22]. In this paper we will consider the more general case in which different initial conditions are assigned to different VEV's, and the system is described by N_f coupled differential equations. Taking for illustration the case $N_f = 2$, we will have to solve the equations

$$\begin{aligned} &\ddot{q}_1 + 3H\dot{q}_1 - d \cdot \, q_1 \, \frac{\Lambda^{2a}}{(q_1 q_2)^{2dN_c}} \, \left[2 + N_c \frac{q_2^2}{q_1^2} \right] = 0 \ , \\ &\ddot{q}_2 + 3H\dot{q}_2 - d \cdot \, q_2 \, \frac{\Lambda^{2a}}{(q_1 q_2)^{2dN_c}} \, \left[2 + N_c \frac{q_1^2}{q_2^2} \right] = 0 \ , \end{aligned} \tag{5}$$

with $H^2 = 8\pi/3M_P^2\ (\rho_m + \rho_r + \rho_Q)$, where M_P is the Planck mass, $\rho_{m(r)}$ is the matter (radiation) energy density, and $\rho_Q = 2(\dot{q}_1^2 + \dot{q}_2^2) + v(q_1, q_2)$ is the total field energy.

2.3. The tracker solution

In analogy with the one-scalar case, we look for power-law solutions of the form

$$q_{tr,i} = C_i \cdot t^{p_i}\ , \qquad\qquad i = 1, \cdots,\ N_f\ . \tag{6}$$

It is straightforward to verify that – when $\rho_Q \ll \rho_B$ – the only solution of this type is given by

$$p_i \equiv p = \frac{1-r}{2}\ , \qquad C_i \equiv C = \left[X^{1-r}\ \Lambda^{2(3-r)}\right]^{1/4}\ , \qquad i = 1, \cdots,\ N_f\ ,$$

with

$$X \equiv \frac{4\ m\ \ (1+r)}{(1-r)^2\ [12 - m(1+r)]}\ ,$$

where we have defined $r \equiv N_f/N_c\ (= 1/N_c, \ldots, 1 - 1/N_c)$. This solution is characterized by an equation of state

$$w_Q = \frac{1+r}{2} w_B - \frac{1-r}{2}\ . \tag{7}$$

Eq. (7) can be derived as usual from energy conservation *i.e.* $d\ (a^3 \rho_Q) = -3\ a^2 p_Q$.

Following the same methods employed in Ref.[17] one can show that the above solution is the unique stable attractor in the space of solutions of eqs. (5). Then, even if the q_i's start with different initial conditions, there is a region in field configuration space such that the system evolves towards the equal fields solutions (6), and the late-time behavior is indistinguishable from the case considered in Ref.[26].

The field energy density grows with respect to the matter energy density as

$$\frac{\rho_Q}{\rho_m} \sim a^{\frac{3(1+r)}{2}}, \tag{8}$$

where a is the scale factor of the universe. The scalar field energy will then eventually dominate and the approximations leading to the scaling solution (6) will drop, so that a numerical treatment of the field equations is mandatory in order to describe the phenomenologically relevant late-time behavior.

The scale Λ can be fixed requiring that the scalar fields are starting to dominate the energy density of the universe today and that both have already reached the tracking behavior. The two conditions are realized if

$$v(q_0) \simeq \rho^0_{crit}\ , \qquad\qquad v''(q_0) \simeq H_0^2\ , \tag{9}$$

where $\rho^0_{crit} = 3M_P^2 H_0^2/8\pi$ and q_0 are the present critical density and scalar fields VEV respectively. Eqs. (9) imply

$$\frac{\Lambda}{M_P} \simeq \left[\frac{3}{4\pi}\frac{(1+r)(3+r)}{(1-r)^2}\frac{1}{rN_c}\right]^{\frac{1+r}{2(3-r)}} \left(\frac{1}{2rN_c}\frac{\rho^0_{crit}}{M_P^4}\right)^{\frac{1-r}{2(3-r)}} \quad , \tag{10}$$

$$\frac{q_0^2}{M_P^2} \simeq \frac{3}{4\pi}\frac{(1+r)(3+r)}{(1-r)^2}\frac{1}{rN_c} \ . \tag{11}$$

Depending on the values for N_f and N_c, Λ and q_0/Λ assume widely different values. Λ takes its lowest possible values in the $N_c \to \infty$ (N_f fixed) limit, where it equals $4 \cdot 10^{-2}(h^2/N_f^2)^{1/6}$ GeV (we have used $\rho^0_{crit}/M_P^4 = (2.5 \cdot 10^{-31} h^{1/2})^4$). For fixed N_c, instead, Λ increases as N_f goes from 1 to its maximum allowed value, $N_f = 1 - N_c$. For $N_c \gtrsim 20$ and N_f close to N_c, the scale Λ exceeds M_P.

The accuracy of the determination of Λ given in (10) depends on the present error on the measurements of H_0, *i.e.,* typically, $\delta\Lambda/\Lambda = \frac{1-r}{3-r}\,\delta H_0/H_0 \lesssim 0.1$.

In deriving the scalar potential (2) and the field equations (5) we have assumed that the system is in the weakly coupled regime, so that the canonical form for the Kähler potential may be considered as a good approximation. This condition is satisfied as long as the fields' VEVs are much larger than the non-perturbative scale Λ. From eqs. (10) and (11), one can compute the ratio between the VEVs today and Λ, and see that it is greater than unity for any N_f as long as $N_c \lesssim 20$.

2.4. Interaction with other cosmological fields

The superfields Q_i and $\overline{Q}_i$ have been taken as singlets under the SM gauge group. Therefore, they may interact with the visible sector only gravitationally, *i.e.* via non-renormalizable operators suppressed by inverse powers of the Planck mass, of the form

$$\int d^4\theta \; K^j(\phi_j^\dagger, \phi_j) \;\cdot \beta^{ji}\left[\frac{Q_i^\dagger Q_i}{M_P^2}\right] \quad , \tag{12}$$

where ϕ_j represents a generic standard model superfield. From (11) we know that today the VEV's q_i are typically $O(M_P)$, so there is no reason to limit ourselves to the contributions of lowest order in $|Q|^2/M_P^2$. Rather, we have to consider the full (unknown) functions β^{ji} and the analogous $\overline{\beta}$'s for the $\overline{Q}_i$'s. Moreover, the requirement that the scalar fields are on the tracking solution today, Eqs. (9) implies that their mass is of order $\sim H_0^2 \sim 10^{-33}$ eV.

The exchange of very light fields gives rise to long-range forces which are constrained by tests on the equivalence principle, whereas the time dependence of the VEV's induces a time variation of the SM coupling constants[30,33]. These kind of considerations sets stringent bounds on the first derivatives of the β^{ji}'s and $\overline{\beta}^{ji}$'s *today,*

$$\alpha^{ji} \equiv \left.\frac{d \log \beta^{ji}\left[x_i^2\right]}{d x_i}\right|_{x_i=x_i^0}, \qquad \overline{\alpha}^{ji} \equiv \left.\frac{d \log \overline{\beta}^{ji}\left[x_i^2\right]}{d\, x_i}\right|_{x_i=x_i^0},$$

where $x_i \equiv q_i/M_P$. To give an example, the best bound on the time variation of the fine structure constant comes from the Oklo natural reactor. It implies that $|\dot{\alpha}/\alpha| < 10^{-15}$ yr^{-1} (see Ref.[34]), leading to the following constraint on the coupling with the kinetic terms of the electromagnetic vector superfield V,

$$\alpha^{Vi},\ \overline{\alpha}^{Vi} \ \lesssim\ 10^{-6}\, \frac{H_0}{\langle \dot{q}_i \rangle}\, M_P\,, \tag{13}$$

where $\langle \dot{q}_i \rangle$ is the average rate of change of q_i in the past 2×10^9yr.

Similar –although generally less stringent– bounds can be analogously obtained for the coupling with the other standard model superfields[33]. Therefore, in order to be phenomenologically viable, any quintessence model should postulate that all the unknown couplings β^{ji}'s and $\overline{\beta}^{ji}$'s have a common minimum close to the actual value of the q_i's.[a]

The simplest way to realize this condition would be via the *least coupling principle* introduced by Damour and Polyakov for the massless superstring dilaton in Ref.[32], where a universal coupling between the dilaton and the SM fields was postulated. In the present context, we will invoke a similar principle, by postulating that $\beta^{ji} = \beta$ and $\overline{\beta}^{ji} = \overline{\beta}$ for any SM field ϕ_j and any flavor i. For simplicity, we will further assume $\beta = \overline{\beta}$.

The decoupling from the visible sector implied by bounds like (13) does not necessarily mean that the interactions between the quintessence sector and the visible one have always been phenomenologically irrelevant. Indeed, during radiation domination the VEVs q_i were typically $\ll M_P$ and then very far from the postulated minimum of the β's. This leads, for the quintessence fields, to the generation of SUSY breaking masses (proportional to H) by the same mechanism discussed by Dine, Randall, and Thomas in Ref.[35].

The main phenomenological effect of these time-dependent SUSY breaking masses is to prevent the fields from taking too large values. This results in an improved attraction to the tracker solution (for further details on this point see Ref.[14]).

2.5. *Numerical Results*

The general results of the previous discussion are illustrated in the figures for the particular case $N_f = 2$, $N_c = 6$.

---

[a] An alternative way to suppress long-range interactions, based on an approximate global symmetry, was proposed in Ref.[30].

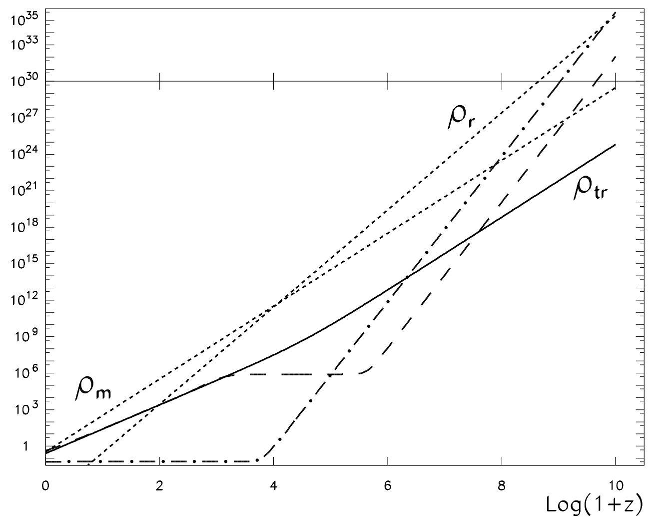

Fig. 1. The evolution of the energy densities ρ of different cosmological components is given as a funcion of red-shift. All the energy densities are normalized to the present critical energy density ρ^0_{cr}. Radiation and matter energy densities are represented by the short-dashed lines, whereas the solid line is the energy density of the tracker solution discussed in Section 2.3. The long-dashed line is the evolution of the scalar field energy density for a solution that reaches the tracker before the present epoch; while the dash-dotted line represents the evolution for a solution that overshoots the tracker to such an extent that it has not yet had enough time to re-join the attractor.

In Fig.1 the energy densities *vs.* redshift are given. We have chosen the same initial conditions for the two VEVs, in order to effectively reproduce the one-scalar case of eq. (4), already studied in Refs.[16,17,22]. No interaction with other fields of the type discussed in the previous section has been considered.

We see that, depending on the initial energy density of the scalar fields, the tracker solution may (long dashed line) or may not (dash-dotted line) be reached before the present epoch. The latter case corresponds to the overshoot solutions discussed in Ref.[22], in which the initial scalar field energy is larger than ρ_B and the fields are rapidly pushed to very large values. The undershoot region, in which the energy density is always lower than the tracker one, corresponds to $\rho^0_{cr} \leq \rho^{in}_Q \leq \rho^{in}_{tr}$. Thus, all together, there are around 35 orders of magnitude in ρ^{in}_Q at redshift $z+1 = 10^{10}$ for which the tracker solution is reached before today. Cleary, the more we go backwards in time, the larger is the allowed initial conditions range.

Next, we explore to which extent the two-field system that we are considering behaves as a one scalar model with inverse power-law potential. In Fig. 2 we plot solutions with the same initial energy density but different ratios between the initial values of the two scalar fields. Given any initial energy density such that – for $q^{in}_1/q^{in}_2 = 1$ – the tracker is joined before today, there always exists a limiting value for the fields' difference above which the attractor is not reached in time.

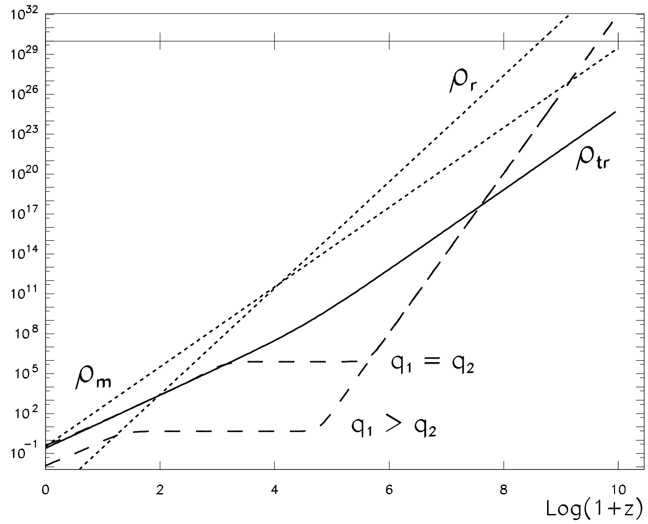

Fig. 2. The effect of taking different initial conditions for the fields, at the same initial total field energy. Starting with $q_1^{in}/q_2^{in} = 10^{15}$ the tracker behaviour is not reached today. For comparison we plot the solution for $q_1^{in}/q_2^{in} = 1$.

3. Aknowledgments

We thank Massimo Pietroni with whom we obtained most of the results reported in this talk. We are also grateful to the organizer of this Workshop, Prof. Milla Baldo Ceolin, for the nice and stimulating atmosphere in which the meeting took place.

1. S.Perlmutter *et al.*, *astro-ph/9812133*; S.Perlmutter *et al.*, *astro-ph/9812473*; S.Perlmutter *et al.*, *Nature* **391**, 51 (1998); See also http://www-supernova.LBL.gov/.
2. A.G.Riess *et al.*, *astro-ph/9805201*; A.V.Filippenko and A.G.Riess, *Phys. Rept.* **307**, 31 (1998); A.V.Filippenko and A.G.Riess, *astro-ph/9905049*; P.M.Garnavich *et al*, *astro-ph/9806396*; B.Leibundgut, G.Contardo, P.Woudt, and J.Spyromilio, *astro-ph/9812042*. See also http://cfa-www.harvard.edu/cfa/oir/Research/supernova/HighZ.html.
3. J.G.Bartlett, A.Blanchard, M.Le Dour, M.Douspis, and D.Barbosa, *astro-ph/9804158*; G.Efstathiou, *astro-ph/9904356*; G.Efstathiou, S.L.Bridle, A.N.Lasenby, M.P.Hobson, and R.S.Ellis, *astro-ph/9812226*; C.Lineweaver, *astro-ph/9805326*.
4. R.G.Carlberg, H.K.C.Yee, and E.Ellingson, *Astrophys. J.* **478**, 462 (1997); J.Carlstrom, 1999, *Physica Scripta*, in press.
5. See for example: S.M.Carroll, W.H.Press and E.L.Turner, *Annu. Rev. Astron.*

Astrophys. **30**, 499 (1992)

6. R.R.Caldwell, R.Dave, and P.J.Steinhardt, *Phys. Rev. Lett.* **80**, 1582 (1998).
7. M.S.Turner, *astro-ph/9904051*.
8. Q. Shafi and F.W. Stecker, *Phys. Lett. B* **53** (1984) 1292; S.A. Bonometto and R. Valdarnini, *Astroph. J.* **299** (1985) L71; S. Achilli, F. Occhionero and R. Scaramella, *Astroph. J.* **299** (1985) L77; J.A. Holtzman, *Astroph. J. Suppl.* **71** (1981) 1; A.N.Taylor and M. Rowan-Robinson, *Nature* **359** (1992) 396; J.A. Holtzman and J. Primack, *Astroph. J.* **396** (1992) 113; D. Pogosyan and A. Starobinski, *Astroph. J.* **447** (1995) 465; A. Klypin, J. Holtzman, J. Primack and E. Regos, *Astroph. J.* **415** (1993) 1.
9. H.B.Kim and J.E.Kim, *Nucl. Phys.* **B433**, 421 (1995).
10. A.Masiero, D.Montanino and M.Peloso, *hep-ph/9902380*.
11. S.Colombi, S.Dodelson and L.M.Widrow, *astro-ph/9505029*.
12. E.Pierpaoli, S.Borgani, A.Masiero and M.Yamaguchi, *Phys. Rev.* **D 57**, 2089 (1998).
13. M.S.Turner and M.White, *Phys. Rev.* **D56**, 4439 (1997); T. Chiba, N. Sugiyama and T. Nakamura, *MNRAS* vol. **289**, L5 (1997).
14. A.Masiero, M.Pietroni and F.Rosati, *hep-ph/9905346*.
15. J.A.Frieman and I.Waga, *Phys. Rev.* **D57**, 4642 (1998).
16. P.J.E.Peebles and B.Ratra, *Astrophys. Jour.* **325**, L17 (1988); B.Ratra and P.J.E.Peebles, *Phys. Rev.* **D37**, 3406 (1988).
17. A.R.Liddle and R.J.Scherrer, *Phys. Rev.* **D59**, 023509 (1999).
18. C.Wetterich, *Nucl. Phys.* **B302**, 668 (1988).
19. E.J.Copeland, A.R.Liddle, and D.Wands, *Phys. Rev.* **D57**, 4686 (1998).
20. P.G.Ferreira and M.Joyce, *Phys. Rev. Lett.* **79**, 4740 (1997); P.G.Ferreira and M.Joyce, *Phys. Rev.* **D58**, 023503 (1998).
21. I.Zlatev, L.Wang, and P.J.Steinhardt, *Phys. Rev. Lett.* **82**, 896 (1999).
22. P.J.Steinhardt, L.Wang, and I.Zlatev, *Phys. Rev.* **D59**, 123504 (1999).
23. C.Baccigalupi and F.Perrotta, *Phys. Rev.* **D59**, 123508 (1999); W.Hu, D.J.Eisenstein, M.Tegmark, and M.White, *Phys. Rev.* **D59**, 023512 (1999); A.R.Cooray and D.Huterer, *astro-ph/9901097*; L.Wang and P.J.Steinhardt, *Astrophys. Jour.* **508**, 483 (1998); L.Hui, *astro-ph/9902275*; B.Ratra, R.Stompor, K.Ganga, G.Rocha, N.Sugiyama, and K.M.Górski, *astro-ph/9901014*; C.van de Bruck and W.Priester, *astro-ph/9810340*; J.S.Alcaniz and J.A.S.Lima, *astro-ph/9902298*.
24. L.Wang, R.R.Caldwell, J.P.Ostriker, and P.J.Steinhardt, *astro-ph/9901388*; G.Huey, L.Wang, R.Dave, R.R.Caldwell, and P.J.Steinhardt, *Phys. Rev.* **D59**, 063005 (1999); S.Perlmutter, M.S.Turner, and M.White, *astro-ph/9901052*; T. Chiba, N. Sugiyama and T. Nakamura, *MNRAS* vol. **301**, 72 (1998).
25. D.Huterer and M.S.Turner, *astro-ph/9808133*; T.Nakamura,T.Chiba, astro-ph/9810447, *MNRAS* in press.; T.Chiba,T.Nakamura, *Prog.Theor.Phys.* vol.

100 1077 (1998).
26. P.Binétruy, *hep-ph/9810553*.
27. J.A.Frieman, C.T.Hill, A.Stebbins, and I.Waga, *Phys. Rev. Lett.* **75**, 2077 (1995); K.Choi, *hep-ph/9902292*; J.E.Kim, *hep-ph/9811509*.
28. C.Kolda and D.H. Lyth, *hep-ph/9811375*.
29. P.Brax and J.Martin, *astro-ph/9905040*.
30. S.M.Carroll, *Phys. Rev. Lett.* **81**, 3067 (1998).
31. T.R.Taylor, G.Veneziano, and S.Yankielowicz, *Nucl. Phys.* **B218**, 493 (1983); I.Affleck, M.Dine, and N.Seiberg, *Phys. Rev. Lett.* **51**, 1026 (1983), *Nucl. Phys.* **B241**, 493 (1984). For a pedagogical introduction see also: M.E.Peskin, *hep-th/9702094*, TASI 96 lectures.
32. T.Damour and A.M.Polyakov, *Nucl. Phys.* **B423**, 532 (1994).
33. T.Damour, *gr-qc/9606079*, lectures given at Les Houches 1992, SUSY95 and Corfù 1995.
34. T.Damour and F.Dyson, *Nucl. Phys.* **B480**, 37 (1996).
35. M.Dine, L.Randall, and S.Thomas, *Phys. Rev. Lett.* **75**, 398 (1995).